# Interaction of the transfer processes in semitransparent liquid droplets

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**Abstract**

The study presents the mathematical model of unsteady heat transfer in evaporating semitransparent droplets of non-isothermal initial state and the numerical research method, evaluating selective radiation absorption and its influence on the interaction of transfer processes. The relation of the transfer processes inside droplets and in their surroundings and the necessity of thorough research of these processes are substantiated. When modeling the combined energy transfer in water droplets, the evaluation of thermoconvective stability in evaporating semitransparent liquid droplets is presented; the influence of the droplet initial state on its heating and evaporation process is investigated. The influence of heat transfer peculiarities on the change of the evaporating droplet state is indicated. Main parameters, which decide the peculiarities of the interaction of unsteady transfer processes in droplets and their surroundings, are discussed. The results of the numerical research are compared to the known results of the experimental studies of water droplet temperature and evaporation rate.
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## 1. Introduction

An intensive energy and mass interphase transfer occurs in high temperature two-phase systems ''gas-droplets'' due to big potential difference between the carrying and condensed discrete mediums and highly developed interphase contact surface. Energy and mass transfer processes are distinctly unsteady. The intensity of the system state variation is decided by energy needs for droplet heating and vaporizing. Droplet evaporation intensity depends on nature of liquid, the dispersity of sprayed liquid and the peculiarities of external energetic influence, which are conditioned by the interaction of transfer processes in the droplet and its surroundings. Combined heat transfer occurs in the droplets, located in radiant environment: heat in such droplets is transferred by convection, conduction and radiation. The interaction between radiation and other energy transfer processes is highly intensive in semitransparent liquids and has a number of peculiarities, if compared to a gaseous

environment. First of all, physics of the influence of the total heat flux density components is different. Energy of convective heating is supplied to the droplet surface, while radiant heat flux is being absorbed in the droplet volume, and heats the internal droplet layers. Local non-isothermalities of the temperature field are formed; therefore the intensive redistribution of energy by conductive heat takes place in a semitransparent droplet; as well as by convection, as liquid circulates. The rate of change of the droplet surface temperature is conditioned by the interaction of energy fluxes on the interphase contact surface. This interaction changes the intensity of energy fluxes and the ratio of energy fluxes that heat the droplet and evaporate it. The essential peculiarity of the interaction of combined transfer process in droplets is that not only the dynamics of the droplet state variation is changed, but also the intensities of phase transformations and the external energy flux are changed as well. Hence, the transfer processes in droplets and in their surroundings are closely related. Correct investigation of these processes requires a simultaneous analysis of the regularities of these transfer processes. Therefore the formulation of the ''droplet problem''—''to investigate the interaction of transfer processes in droplets and in their surroundings'' can be called a classical one.

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**Nomenclature**

| | |
|---|---|
| $a$ | thermal diffusivity |
| $B$ | Spalding transfer number |
| $c_p$ | mass specific heat |
| $C_D$ | droplet drag coefficient |
| $C_F$ | friction drag coefficient |
| $D$ | mass diffusivity |
| $g$ | evaporation rate |
| $G$ | mass flow rate |
| $Gr$ | Grashof number |
| $H$ | mass |
| $I$ | control time index |
| $k_c^-$ | effective conductivity parameter |
| $k_\omega$ | spectral index of absorption |
| $L$ | latent heat of evaporation |
| $m$ | mass flux density |
| $M$ | molecular mass |
| $n$ | number of the term in infinite sum |
| $N$ | number of terms in infinite sum |
| $Nu$ | Nusselt number |
| $p$ | pressure |
| $Pe$ | Peclet number |
| $Pr$ | Prandtl number |
| $q$ | heat flux density |
| $r$ | coordinate of a droplet |
| $r_\omega$ | spectral index of light reflection |
| $R$ | radius of a droplet |
| $Ra$ | Rayleigh nuumber |
| $Re$ | Reynolds number |
| $R_*$ | universal gas constant |
| $t$ | dimensionless time |
| $T$ | temperature |
| $w$ | velocity |

*Greek symbols*

| | |
|---|---|
| $\Delta w$ | slip velocity of a droplet in gas |
| $\beta$ | Brewsters angle |
| $\eta$ | dimensionless coordinate |
| $\varphi$ | angle of light beam descent |
| $\lambda$ | thermal conductivity |
| $\mu$ | dynamics viscosity |
| $v$ | kinematics viscosity |
| $\rho$ | density |
| $\tau$ | time |
| $\omega$ | wave number |

*Subscripts*

| | |
|---|---|
| c | convective |
| C | droplet center |
| e | stabilised evaporation |
| f | evaporation |
| g | gas |
| gv | gas–vapor mixture |
| i | time index in a digital scheme |
| I | index of control time |
| j | index of droplet cross-section |
| J | index of droplet surface |
| m | mass average |
| l | conductive |
| L | liquid |
| r | radiation |
| R | droplet surface |
| v | vapor |
| $\omega$ | spectral |
| $\Sigma$ | total |
| 0 | initial state |
| $\infty$ | far from a droplet |

*Superscripts*

| | |
|---|---|
| k | number of iteration |
| kr | critical |
| * | variable |
| "+" | external side of a surface |
| "−" | internal side of a surface |

History of the researches of the "droplet problem"' is more than 100 years old. The beginner of the droplet research was J.C. Maxwell, who determined the decrease of evaporating droplet temperature in 1877 [1]. In the earlier studies the main attention was paid to the investigations of external heat and mass transfer in steady-still pure liquid droplets in low-temperature surroundings. These investigations are thoroughly discussed in [2]. Later "droplet" investigations include the effect of droplet slip in gas; the influence of evaporation on heating intensity; high temperature; combustion and other factors. These studies are discussed in [3–7] and other reviews. The attention to theoretical and experimental research of "droplet" has not decreased later as

well [8–23]. The "droplet" investigations remain relevant in our days as well due to wide technological applications of the two-phase systems "droplet-gas" and due to the necessity to understand deeper the interaction of unsteady combined transfer processes.

Theoretical investigations of the "droplet problem" mostly often are carried out using the premises that allow avoiding the necessity to evaluate the interaction of radiation and convection in droplets. When the effect of radiation is neglected, the influence of liquid circulation on heat transfer in droplets is evaluated thoroughly [8,24,25]. The interaction of unsteady radiant-conductive energy transfer processes is thoroughly evaluated when the influence of liquid circulation is cancelled [17].

The results of experimental research regarding liquid stability in symmetric volumes are presented in the studies [26,27]. The methods for calculation of radiation [17,28–37] allow evaluating the influence of spectral optical characteristics on combined energy transfer in semitransparent liquid droplets. The method presented in [10,38] allows considering the selectivity of radiation in droplet surroundings. The regularities of convective heating of a solid sphere are thoroughly examined. Since the interaction of radiation and convection transfer processes in gas is relatively negligible [39], the intensity of convective heating of evaporating droplets is mostly often calculated by well-known empiric expressions for a non-evaporating sphere and by introducing the corrections that evaluate the Stefan flow. Despite of the fact that the "droplet problem" is quite deeply analyzed, the methods that evaluate the interaction of combined transfer processes are not completely finished; therefore further thorough "droplet" researches have to be performed.

This study presents the modeling of unsteady heat transfer in evaporating droplets of a non-isothermal initial state.

## 2. Problem formulation

The interaction of combined transfer processes in evaporating liquid droplets can be evaluated, taking into account well-analyzed regularities of convective, radiant, conductive and radiant-conductive heat transfer. The change of a droplet state in a particular finite period of time is intended to be calculated by the model of radiant-conductive heat transfer and then, at the end of this period, to specify the parameters of transfer processes after additional evaluation of heat influence on the dynamics of the droplet state. In this case it is impossible to solve the problem of radiant-conductive heat transfer in a droplet, using solid numerical scheme, because the initial conditions of separate finite time intervals will be changed. The droplet temperature field, changed due to liquid circulation, will decide these conditions. Hence, the droplet can be non-isothermal in every interval of $\Delta\tau_i$ variation, excluding the first interval. Therefore, first of all it is necessary to construct a mathematical model of unsteady radiant-conductive heat transfer in a droplet of non-isothermal initial state.

The intensity of temperature field and conductive heat flux variation in the case of radiant-conductive heat transfer in a spherically symmetrical droplet of semi-transparent liquid is defined by the following energy equation:

$$\rho c_p \frac{\partial T(r,\tau)}{\partial \tau} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\lambda\frac{\partial T(r,\tau)}{\partial r}\right] + \frac{1}{r^2}[r^2 q_r(r,\tau)]. \quad (1)$$

When the unambiguity conditions of Eq. (1) are formulated for the initially isothermal droplet, the change of surface temperature of which is defined by a known time function $T(R,\tau) = T_R(\tau)$, the shape of the transfer problem with a defined energy source can be assigned to the problem of radiant-conductive heat transfer and thus to apply its formal solution, having the form of an integral equation [40]. Such method of combined heat transfer has been successfully used when describing the unsteady temperature field in the droplet, which warms under the influence of monochromatic radiation [40], or in the case of the function $q_r(r,\tau)$ that defines the absorption of any radiation in the droplet [17].

When at the beginning of heating process the droplet is non-isothermal, its initial state is defined by a certain coordinate function $T(r,\tau) = T_0(r)$. In the case of symmetrical heat transfer in the droplet, this function has to satisfy the following condition:

$$\left.\frac{dT_0(r)}{dr}\right|_{r=0} = 0. \quad (2)$$

Time intervals $\Delta\tau_i$ will be selected in such a way that the droplet diameter will change during them insignificantly, therefore it is possible to state that the diameter will be constant. At the end of time change interval $\Delta\tau_i$ the droplet size will change in steps, proportionally to the evaporation rate [17]:

$$R^2 = R_0^2 - 2\frac{\Delta\tau_i D M_v}{\rho_L R_* T_{gv,R}}\left[p_{v,R} - p_{v,\infty} + \left(\frac{M_v}{M_g}\right)\right.$$
$$\left. \times\left(p\ln\frac{p - p_{v,\infty}}{p - p_{v,R}} - p_{v,R} + p_{v,\infty}\right)\right]. \quad (3)$$

Temperature of liquid vapor and gas mixture in the vicinity of the droplet $T_{vg,R}$ is close to the droplet surface temperature $T_R$. Instant values of the latter temperature and the rate of its variation is decided by the transfer process interaction. At every time instant the temperature of the droplet surface is such that the equality of the energy fluxes taken to the surface and taken away from it is ensured:

$$\vec{q}_\Sigma^+ + \vec{q}_\Sigma^- + \vec{q}_f^+ = 0. \quad (4)$$

Under the statement that the dynamics of the droplet surface temperature variation is known, the unambiguity conditions of Eq. (1) in the case of the constant size droplet of initially non-isothermal state can be defined as

$$T(r,0) = T_0(r); \quad T(R,0) = T_{R,0};$$
$$T(R,\tau) = T_R(\tau), \quad R(\tau) = R_0. \quad (5)$$

Using dimensionless variables:

$$\eta = \frac{r}{R}; \quad t = \frac{a_0 \tau}{R_0}; \quad \overline{T}(\eta, t) = \frac{T(r, \tau)}{T_{R,0}};$$
$$\overline{q}_r(\eta, t) = \frac{R_0 q_r(r, \tau)}{T_{R,0} \lambda_0} \tag{6}$$

and stating that $\lambda(r) \cong$ const, Eq. (1) can be defined in the following way:

$$\frac{\partial \overline{T}(\eta, t)}{\partial t} = \frac{a}{a_0} \left[ \frac{\partial^2 \overline{T}(\eta, t)}{\partial \eta^2} + \frac{2}{\eta} \frac{\partial \overline{T}(\eta, t)}{\partial \eta} \right] + \frac{\rho c_p}{\rho_0 c_{p0}} \frac{1}{\eta^2}$$
$$\times [\eta^2 \overline{q}_r(\eta, t)]. \tag{7}$$

Use of additional function

$$\Theta(\eta, t) = \eta [\overline{T}(\eta, t) - \overline{T}_R(t) \overline{T}_0(\eta)] \tag{8}$$

allows to give a new shape to Eq. (7):

$$\frac{\partial \Theta(\eta, t)}{\partial t} = a_1^2 \frac{\partial^2 \Theta(\eta, t)}{\partial \eta^2} + f_s(\eta, t) \tag{9}$$

here $a_1^2 \equiv a/a_0$; while the source function

$$f_s(\eta, t) = \frac{a}{a_0} \frac{\overline{T}_R(t)}{\eta} \frac{\partial}{\partial \eta} \left[ \eta^2 \frac{\partial \overline{T}_0(\eta)}{\partial \eta} \right] + \frac{\rho_0 c_{p0}}{\rho c_p} \frac{1}{\eta} \frac{\partial}{\partial \eta}$$
$$\times [\eta^2 \overline{q}_r(\eta, t)] - \overline{T}_0(\eta) \frac{\partial \overline{T}_R(t)}{\partial t} \tag{10}$$

evaluates the influence of radiation absorption in a droplet, its initial non-isothermality and surface temperature variation on the change of the droplet state. The source function $f_s(\eta, t)$ can be determined as a definite one, since the derivatives in this function are determined by the initial and boundary conditions (5); the function of radiant flux $\overline{q}_r(\eta, t)$ in the case of iterative calculations can be calculated according to the temperature field, adjusted in the previous iteration. Since the function $f_s(\eta, t)$ is defined and unambiguity conditions for Eq. (9) are indiscrete

$$\Theta(0, t) = 0; \quad \Theta(1, t) = 0; \quad \Theta(\eta, 0) = 0, \tag{11}$$

the system of Eqs. (9)–(11) fulfills all conditions of the Direchle transfer problem. That allows us to express the function $\Theta(\eta, t)$ in the following way:

$$\Theta(\eta, t) = 2 \sum_{n=1}^{\infty} \sin(n\pi\eta) \int_0^t \exp[-(n\pi a_1)^2 (t - t^*)]$$
$$\times \int_0^1 \left\{ a_1^2 \overline{T}_R(t) \frac{\sin(n\pi\eta)}{\eta} \frac{\partial}{\partial \eta} \left[ \eta^2 \frac{\partial \overline{T}_0(\eta)}{\partial \eta} \right] \right.$$
$$+ \frac{\rho_0 c_{p0}}{\rho c_p} \frac{\sin(n\pi\eta)}{\eta} \frac{\partial}{\partial \eta} [\eta^2 \overline{q}_r(\eta, t^*)]$$
$$\left. - \overline{T}_0(\eta) \eta \sin(n\pi\eta) \frac{\partial \overline{T}_R(t^*)}{\partial t^*} \right\} d\eta \, dt^*. \tag{12}$$

By integrating the expression (12) by parts, we receive the below presented integral equation, describing the unsteady temperature field:

$$\overline{T}(\eta, t) = \overline{T}_R(t) \overline{T}_0(\eta) + \frac{2}{\eta} \sum_{n=1}^{\infty} \sin(n\pi\eta) \int_0^t f_n(\eta, t^*)$$
$$\times \exp[-(n\pi a_1)^2 (t - t^*)] dt^*, \tag{13}$$

in which the function of the infinite sum number

$$f_n(\eta, t^*) = \frac{(-1)^n}{n\pi} \overline{T}_0(\eta) \frac{d\overline{T}_R(t^*)}{dt^*} + \int_0^1 \left[ \frac{\rho_0 c_{p0}}{\rho c_p} \overline{q}_r(\eta, t^*) \right.$$
$$\left. + a_1^2 \overline{T}_R(t^*) \frac{d\overline{T}_0(\eta)}{d\eta} \right] [\sin(n\pi\eta) - \cos(n\pi\eta) n\pi\eta] d\eta \tag{14}$$

evaluates the peculiarities of radiation absorption in the droplet and its temperature variation, and the influence of droplet initial non-isothermality on the process of droplet heating. The temperature of heating droplet center is defined by the equation:

$$\overline{T}(0, t) = \overline{T}_R(t) \overline{T}_0(\eta) + 2\pi \sum_{n=1}^{\infty} n \int_0^t f_n(\eta, t^*)$$
$$\times \exp[-(n\pi a_1)^2 (t - t^*)] dt^*. \tag{15}$$

When $\eta = 1$, $\overline{T}(1, t) = \overline{T}_R(t)$. After differentiation of the expression (13) along the coordinate, the equation of a local temperature gradient in the droplet is obtained:

$$\frac{\partial \overline{T}(\eta, t)}{\partial \eta} = \overline{T}_R(t) \frac{\partial \overline{T}_0(\eta)}{\partial \eta} + 2 \sum_{n=1}^{\infty} \left[ \frac{n\pi}{\eta} \cos(n\pi\eta) \right.$$
$$\left. - \frac{1}{\eta^2} \sin(n\pi\eta) \right] \int_0^t f_n(\eta, t^*)$$
$$\times \exp[-(n\pi a_1)^2 (t - t^*)] dt^*. \tag{16}$$

Eq. (16) allows calculating a local total flux in a semi-transparent droplet in the case of combined radiant-conductive energy transfer, if radiant heat flux density in the droplet is known:

$$q_\Sigma(r, \tau) = q_r(r, \tau) + q_l(r, \tau)$$
$$= q_r(r, \tau) + \lambda_L(r, \tau) \frac{T_{R0}}{R_0} \frac{\partial \overline{T}(\eta, t)}{\partial \eta}. \tag{17}$$

Total heat flux density on the droplet surface:

$$q_\Sigma(R^-, \tau) = q_r(R^-, \tau) + \lambda_L \frac{T_{R0}}{R} \frac{\partial \overline{T}(\eta, t)}{\partial \eta} \bigg|_{\eta=1}, \tag{18}$$

here

$$\frac{\partial \overline{T}(\eta, t)}{\partial \eta} \bigg|_{\eta=1} = \overline{T}_R(t) \frac{\partial \overline{T}_0(\eta)}{\partial \eta} \bigg|_{\eta=1} + 2\pi \sum_{n=1}^{\infty} n(-1)^n$$
$$\times \int_0^t f_n(\eta, t^*) \exp[-(n\pi a_1)^2 (t - t^*)] dt^*. \tag{19}$$

The presented equations (13)–(19) unambiguously describe the unsteady radiant-conductive transfer pro-

cesses in the non-isothermal initial state droplets, when the functions $q_r(r, \tau)$ and $T_R(\tau)$ are determined. These functions define radiation absorption in the droplets and the change of their surface temperature. The change of $T_R(\tau)$ can be concretized by iterative calculations, minimizing Eq. (4), which defines the balance of energy fluxes on the droplet surface. When the droplet temperature field, specified in the previous iteration, is known, the distribution of radiant flux is calculated by the method [17], which evaluates the dependence of spectral optical effects on the droplet surface on its state. Since the spectral indices of radiation absorption of semitransparent liquids are finite, it is possible to state that radiation is not absorbed by the surfaces of semitransparent liquid droplets, i.e. $q_r(R^+, \tau) \cong q_r(R^-, \tau)$; therefore the expression (4) can be written in the following way:

$$q_c^+(\tau) - \lambda_L \frac{T_{R,0}}{R_0} \frac{\partial \overline{T}(\eta, t)}{\partial \eta}\bigg|_{\eta=1} + Lm(\tau) = 0. \tag{20}$$

The following equation defines the intensity of phase transformations in Eq. (20), characterized by liquid vapor flow density on the droplet surface:

$$m = \frac{D}{T_{gv,R}} \frac{M_v}{R_* R} \left[ p_{v,R} - p_{v,\infty} + \left(\frac{M_v}{M_g}\right)\left( p \ln \frac{p - p_{v,\infty}}{p - p_{v,R}} \right.\right.$$
$$\left.\left. - p_{v,R} + p_{v,\infty} \right) \right]. \tag{21}$$

This equation evaluates the influence of the Stefan flow [40].

In the case of droplet evaporation, vapor flow, spreading from an evaporating droplet, resists hot gas to approach to it; therefore thermal boundary layer becomes thicker, and convective heat transfer to the droplet decreases. This effect is evaluated by introducing the function $Nu_L^+ = f(B)Nu$ of the Spalding transfer number $(B)$ into well-known empiric expressions, which describe the intensity of solid sphere convective heating $(Nu)$. These functions usually are expressed by logarithmic or power expressions. The intensity of convective heating of an evaporating sphere in present study was calculated by the expression [9]:

$$(1 + B)^{0.7} Nu_L^+ = 2 + 0.57 Re^{1/2} Pr^{1/3}. \tag{22}$$

$Re = 2R\rho_{g,\infty}\Delta w / \mu_{gv}$; the thermophysical properties of vapor–gas mixture are evaluated at an appropriate reference temperature $T \equiv T_R + 0.5(T_g - T_R)$. In the case of natural convection [19]:

$$Nu = 2 + 0.6 Gr^{1/4} Pr^{1/3}. \tag{23}$$

The Spalding transfer number $B$ depends on liquid nature, the difference between gas and liquid energetic levels and the ratio of energy flux, which together with the convective flux takes place in the process of the droplet evaporation, and the convective heating flux. In the case of equilibrium evaporation of the droplet that heats in radiating medium, the Spalding transfer number is expressed as follows [28,41]:

$$B = \frac{c_{p,vg}(T_g - T_R)}{L} \left[ 1 + \frac{q_r(R^+)}{q_c(R^+)} \right]. \tag{24}$$

During unsteady evaporation the droplet heats up, therefore not all energy, supplied to it, takes place in the evaporation process. Then, calculating the parameter $B$ according to the expression (24), the influence of mass transfer on convective heating intensity would be overestimated. The intensity of energy, used for the droplet heating, is defined by the total heat flux on the internal side of the droplet surface $q_\Sigma(R^-) = q_l(R^-) + q_r(R^-)$. Evaluating that $q_r(R^+, \tau) \equiv q_r(R^-, \tau)$, in the case of radiant-conductive transfer the Spalding transfer number can be described by the following universal expression:

$$B = \frac{c_{p,vg}(T_g - T_R)}{L} \left[ 1 + \frac{q_l(R^-)}{q_c(R^+)} \right], \tag{25}$$

here

$$q_l(R^-) = -\lambda_L \frac{\partial T(r, \tau)}{\partial r}\bigg|_{r=R^-} \tag{26}$$

Physical interpretation of conductivity component $q_l(R^-)$ of the total heat flux in Eq. (25) changes during droplet evaporation process. Three periods of the droplet state change can be selected according to the physical meaning of $q_l(R^-)$ interpretation [17]: the initial, in which $q_l(R^-)$ corresponds to the external convection part together with radiant energy absorbed in the droplet and heating it, the transient, in which $q_l(R^-)$ corresponds to part radiant energy absorbed in the droplet and evaporating it together with convective energy, and the final, in which $q_l(R^-) \equiv q_r(R^-)$, therefore $q_\Sigma(R^-) = 0$ and all energy supplied to the droplet evaporates it.

Dynamics of a droplet, slipping in gas, depends on the droplet size, natures of liquid and gas, as well as on the total drag coefficient:

$$dw_L = \frac{3}{8} \frac{C_D}{R} \frac{\rho_g}{\rho_L} \frac{|w_L - w_g|}{(w_L - w_g)^{-1}}. \tag{27}$$

The value of the droplet total drag coefficient depends on its friction drag coefficient component $C_F$ and pressure drag component $C_p$. Friction drag coefficient is sensitive to the intensity of phase transformation, the influence of which can be evaluated quite effectively by introducing power function of the parameter $B$ into the solid sphere drag expressions [7]:

$$C_F = \frac{14.5}{(1 + B)Re^{0.71}}. \tag{28}$$

Friction drag coefficient of an evaporating droplet is $(1 + B)^{0.6}$ times bigger than that of a solid sphere at the same $Re$ [7]. Furthermore, the value of this coefficient is influenced by the change of gas–vapor mixture physical properties in thermal boundary layer. The influence of all the above-mentioned factors on the total drag coefficient of an evaporating droplet is quite effectively evaluated by calculating $C_D$ according to the expression

$$C_D = \frac{24}{(1 + B)^{0.2}} \frac{1 + 0.2Re^{0.63}}{Re}, \qquad (29)$$

as $10 < Re < 260$ [7].

When modeling combined heat transfer in evaporating droplets according to radiant-conductive heat transfer model, it is necessary to know the boundary conditions, according to which it is possible to neglect the influence of liquid circulation on energy transfer in the droplets. Liquid circulation in the droplet can originate from internal and external factors. Despite of specific factors like the Marangoni effect, possible influence of an electromagnet field, liquid heterogenity, coalescence of droplets, contact with hot surface, etc., the effect of thermogravitational forces, arising due to the non-isothermality of the droplet temperature field, can be considered as the main internal factor. The main external factor is the effect of drag forces, arising at the surface of droplet as it slips in gas. The internal factor influences natural circulation of liquid, while the external factor can start forced liquid circulation in a droplet. The influence of droplet non-isothermality on the intensity of natural liquid circulation is defined by the Rayleigh number ($Ra_L$); the intensity of forced circulation-by the Peclet number ($Pe_L$). The above-mentioned factors can take place, but their effect may be not enough for liquid circulation to begin. The conditions of circulation are described by critical numbers $Ra_L^{kr}$ and $Pe_L^{kr}$. If the value of at least one number is exceeded, liquid will circulate in the droplet.

A particular value of $Ra_L^{kr}$ significantly depends on liquid volume and heat transfer conditions. Six types of liquid circulation [26] can be picked out according to the character of non-isothermality in the volume of spherically symmetrical liquid volume. Every type of circulation has its particular trajectory of liquid particle motion and the particular values of critical $Ra_L^{kr}$. The smallest critical values of $Ra_L^{kr}$ are in the range of hundred, when non-symmetrical heating of spherically symmetrical liquid volume occurs. In such case the liquid droplets move in circular trajectories, which are in vertical plains and which do not pass through the center of sphere. In the case of symmetrical heat transfer the liquid particles move in circular trajectories, in vertical plains along the radius of sphere, and $Ra_L^{kr}$ is in the range of thousand. Minimal values of $Ra_L^{kr}$ are very important when evaluating liquid stability in evaporating droplets. These values depend on the ratio $\bar{k}_\lambda$ of thermal conductivities of liquid and its environment: when $\bar{k}_\lambda \to 0$, $Ra_L^{kr} \approx 780$; when $\bar{k}_\lambda = 1$, $Ra_L^{kr} \approx 470$; when $\bar{k}_\lambda \to \infty$, $Ra_L^{kr} \approx 250$ [26]. The results of theoretical investigations of $Ra_L^{kr}$ quite well match to the results of experimental investigations [27] $Ra_L^{kr} = 350 \pm 30$, as $\bar{k}_\lambda = 3.26$.

The influence of the external factor on the intensity of forced liquid circulation is evaluated by the Peclet number $Pe_L = Re_L Pr_L$. As $Pe_L < 5$, the influence of liquid circulation on energy transfer in the droplet is negligible [8]. Therefore $Pe_L^{kr} \approx 5$. The Reynolds number ($Re_L = 2Rw_{L,R}/v_L$) is calculated by maximum liquid velocity on the droplet surface:

$$w_{L,R} = C_F \frac{\Delta w}{32} \frac{\mu_g}{\mu_L} Re. \qquad (30)$$

In such case, the increase of the convective heart transfer intensity due to liquid circulation, comparing to pure heat conduction, can be defined by a non-linear Peclet number function [8]:

$$\begin{aligned}
\bar{k}_c &= \frac{Nu_L(Pe_L)}{Nu_L(0)} \\
&= 1.86 + 0.86\tanh[2.245\log_{10}(Pe_L/30)]. \qquad (31)
\end{aligned}$$

Hence, in the case of radiant-convective heat transfer in a droplet, the convective component of total heat flux $q_{\Sigma,L}(r,\tau) = q_{r,L}(r,\tau) + q_{c,L}(r,\tau)$ consists of natural and forced convection components of the total mixed convection heat flux. As the intensity of mixed convection (if compared with conductive heat transfer) in the case of symmetric heat transfer in a droplet is defined by a certain function of dimensionless coordinates $f_{c,L}(\eta,t)$, the change of the droplet surface temperature in the case of radiant-convective heat transfer is defined by the following functional:

$$q_c^+(\tau) - \lambda_L \frac{T_{R,0}}{R_0} \left[ f_{c,L}(\eta,t) \frac{\partial \overline{T}_c(\eta,t)}{\partial \eta} \right]\Bigg|_{\eta=1} + Lm(\tau) = 0. \qquad (32)$$

Values in Eq. (32): when $Ra_L < Ra_L^{kr}$ and $Pe_L < Pe_L^{kr}$, $f_{c,L}(1,t) = 1$; when $Ra_L < Ra_L^{kr}$ and $Pe_L > Pe_L^{kr}$, $f_{c,L}(1,t) \equiv k_c^-$.

Then the transfer parameters $\overline{T}_1(\eta,t)$, $\partial \overline{T}_1(\eta,t)/\partial \eta$, $q_1(\eta,t)$, calculated by the expressions (13)–(17) will correspond to such case of radiant-conductive heat transfer, as if they were occurring under the boundary conditions, which would exist in the case of radiant-convective heat transfer. Stating that the function $f_{c,L}(\eta,t)$ that defines the intensity of liquid circulation, is known, the instant temperature field of the droplet in the case of radiant-convective heat transfer is calculated by the expression

$$\overline{T}_c(\eta,t) = \overline{T}_R(t) - \int_\eta^1 \left[ \frac{\partial \overline{T}_1(\eta,t)}{\partial \eta} \Big/ f_{c,L}(\eta,t) \right] d\eta. \qquad (33)$$

## 3. Numerical solution

When constructing a numerical research scheme of combined heat transfer in semitransparent liquid droplets, it is stated that the functions of radiant heat flux $q_r(\eta,t)$, the change of the interphase contact surface temperature $T_R(t)$ and the intensity of convective heat transfer $f_{c,L}(\eta,t)$ are known. The predicted time of the droplet state change $t$ is divided into $I-1$ number of time change intervals $\Delta t_i^* = t_i^* - t_{i-1}^*$ ($t^* = 0$, as $i=1$; $t^* = t$, as $i = I$). Droplet diameter is divided into $J-1$ number of finite intervals $\Delta \eta_j = \eta_j - \eta_{j-1}$ ($\eta = 0$, as $j=1$; $\eta = 1$, as $j = J$). Then the change of the surface temperature of an evaporating droplet, in which energy is being transferred by all heat transfer ways, can be calculated according to the following scheme:

$$Nu_{L,I}^+ \frac{\lambda_{vg,I}(T_g - T_{R,I})}{2R_0} \left\{ 1 + \frac{c_{p,vg,I}(T_g - T_{R,I})}{L} \left[ 1 + \frac{q_{c,I,J}^-}{q_{c,I,J}^+} \right] \right\}^{-0.7}$$

$$+ q_{c,I,J}^- + \frac{D_I}{T_{R,I}} \frac{L_I M_{v,I}}{R_* R_0} \left[ p_{v,R,I} - p_{v,\infty,I} + \left( \frac{M_{v,I}}{M_{g,I}} \right) \right.$$

$$\left. \times \left( p \ln \frac{p - p_{v,\infty,I}}{p - p_{v,R,I}} - p_{v,R,I} + p_{v,\infty,I} \right) \right] = 0, \qquad (34)$$

here

$$q_{c,I,J}^- = -\lambda_{L,I} \frac{T_{R,0}}{R_0} f_{c,L,I,J} \left\{ \overline{T}_{R,I} \frac{\partial \overline{T}_0(\eta)}{\partial \eta} \bigg|_{\eta=1} + 2\pi \sum_{n=1}^{\infty} n(-1)^n \right.$$

$$\times \sum_{i=2}^{I} \frac{1}{(n\pi a_{1,i})^2} \left[ \frac{(-1)^n}{n\pi} \overline{T}_{0,j} \frac{\overline{T}_{R,i} - \overline{T}_{R,i-1}}{t_i - t_{i-1}} + E_{i,J} \right] \right\}$$

$$\times \{\exp[(n\pi a_{1,i})^2(t_i - t_I)] - \exp[(n\pi a_{1,i})^2(t_{i-1} - t_I)]\}. \qquad (35)$$

The parameter $E_{i,J}$ is calculated by a numerical expression, which is constructed similarly to Eq. (34) [17], additionally evaluating the local gradients of the initial temperature field and the peculiarities of dimensionless coordinates. The functional (34) is minimized using the method of fastest descent in regard to the evaporating droplet surface temperature, by ensuring the following condition:

$$\left| 1 - \frac{q_{\Sigma,I}^k(R^+)}{q_{\Sigma,I}^k(R^-) + q_{f,I}^k(R^+)} \right| \cdot 100\% \leqslant 0.01\%. \qquad (36)$$

In the case of radiant-conductive heat transfer of an evaporating droplet, the function $f_{c,L}(1,t) = 1$ in Eq. (34). The temperature field of the droplet and its local gradients are calculated as follows:

$$\overline{T}_{I,j} = \overline{T}_{R,I} \overline{T}_{0,j} + \frac{2}{\eta_j} \sum_{n=1}^{\infty} \sin(n\pi\eta) \sum_{i=2}^{I} \frac{1}{(n\pi a_{1,i})^2}$$

$$\times \left[ \frac{(-1)^n}{n\pi} \overline{T}_{0,j} \frac{\overline{T}_{R,i} - \overline{T}_{R,i-1}}{t_i - t_{i-1}} + E_{i,J} \right]$$

$$\times \{\exp[(n\pi a_{1,i})^2(t_i - t_I)] - \exp[(n\pi a_{1,i})^2(t_{i-1} - t_I)]\}, \qquad (37)$$

$$\frac{\partial \overline{T}}{\partial \eta} \bigg|_{I,j} = \overline{T}_{R,I} \frac{\partial \overline{T}_0}{\partial \eta} \bigg|_j + 2 \sum_{n=1}^{\infty} \left[ \frac{n\pi}{\eta} \cos(n\pi\eta) - \frac{1}{\eta_j^2} \sin(n\pi\eta) \right]$$

$$\times \sum_{i=1}^{I} \frac{1}{(n\pi a_{1,i})^2} \left[ \frac{(-1)^n}{n\pi} \overline{T}_{0,j} \frac{\overline{T}_{R,i} - \overline{T}_{R,i-1}}{t_i - t_{i-1}} + E_{i,J} \right]$$

$$\times \{\exp[(n\pi a_{1,i})^2(t_i - t_I)] - \exp[(n\pi a_{1,i})^2(t_{i-1} - t_I)]\}. \qquad (38)$$

The dimension of the droplet is specified according to the expression (3) at time instant $t_I$. Then the initial and boundary conditions (5) are specified and new time period $(0–t)$, is predicted, during which the changes of the constant size droplet state are calculated again. Similar calculations are performed till the droplet evaporates out completely.

In the case of radiant-convective heat transfer the temperature field of the droplet is specified at time instant $t_I$

$$\overline{T}_{c,I,j} = \overline{T}_{R,I} - 0.5 \sum_{j}^{J-1} \left[ \left( \frac{\partial \overline{T}_{1,I}}{\partial \eta} \bigg|_j \bigg/ f_{c,L,I,j} \right) \right.$$

$$\left. + \left( \frac{\partial \overline{T}_{1,I}}{\partial \eta} \bigg|_{j+1} \bigg/ f_{c,L,I,j+1} \right) \right] (\eta_{j+1} - \eta_j). \qquad (39)$$

After that $R_I$ is calculated similarly to the radiant-conductive heat transfer case; the conditions (5) are specified; new time period is predicted and the changes of the constant size droplet state are calculated again. Similar calculations are performed till the droplet evaporates out completely.

## 4. Results and discussion

The change of the state of semitransparent liquid (water, n-decane, n-heptane) droplet is modeled numerically under different conditions of heat and mass transfer, as the droplets evaporate in dry air flow, moving at velocity $w_g$ and temperature $T_g$ ($\rho_{v,\infty} = 0$). In the case of convective heat transfer the initial velocity of the droplet slip in gas is $\Delta w_0 = |w_{g,0} - w_{L,0}| > 0$; in the case of conductive heating $\Delta w = 0$. When considering combined heating of droplets, it is stated that absolutely black source of radiation exists in medium that surrounds the droplets. The temperatures of the radiation source and gas are equal. Local radiant heat flux in the

droplets is calculated according to the method presented in [17]. Radiation spectrum interval 0.8–800 μm is gradually divided into 150 parts according to the wave number. The intensity of monochromatic radiation according to a radiation spectrum is integrated using the rectangular method, according to the angle of light beam descent—according to the Gauss method, using 5-point scheme. Spectral optical characteristics of water are taken according to [42,43], the characteristics of heptane and decane—according to [44]. Spectral coefficients of light reflection on the droplet surface and Brewster's angles are calculated by the method [45]. The droplet diameter is divided into 60 intervals, making them thinner in the droplet surface layers. 120 members of the infinite sum are evaluated in the expressions of the unsteady transfer parameters. Time change intervals are selected in such way that the mean mass temperature of the droplet would change in not more than 1°; the radius of the droplet would decrease in not more than 1 μm. In such case very high accuracy of the calculation of transfer parameters in evaporating droplets is assured [46].

The interphase contact temperature, described by Eq. (32), has a tremendous influence on the intensity of the transfer process interaction in a two-phase system "droplets-gas". Even small changes of this temperature condition significant changes of energy and mass fluxes on the surface of an evaporating droplet. Therefore it is very important in Eq. (32) to select as accurately as possible the value of the function $f_{c,L}(\eta, t)|_{\eta=1}$ that evaluates the influence of liquid circulation intensity on energy transfer in the droplets. Therefore the evaluation of natural liquid circulation possibility in evaporating droplets under the influence of the Archimedean forces is presented at the beginning.

### 4.1. The evaluation of thermoconvective resistance of evaporating droplets

In this case it is stated that the velocity of droplet slip in gas equals zero, therefore there are no external factors that can initiate forced circulation in evaporating droplets. For the simplicity we make an assumption that the droplets of isothermal initial state get into a gas flow: $\overline{T}_0(\eta) = 1$; $\mathrm{d}\overline{T}_0(\eta)/\mathrm{d}\eta = 0$. In such case the limitation of the droplet size constancy $R(\tau) = R_0$ is not necessary in the unambiguosity conditions (5), and Eq. (13) can be easily transformed into Eq. (18), presented in the study [17], which defines the unsteady temperature field in the droplet with changing diameter in the case of radiant-conductive heat transfer in it. In this case the variation of the evaporating droplet's state change is calculated using the solid scheme of the numerical research.

Resistance of heating and evaporating semitransparent liquid droplets to the increase of thermoconvection inside them was evaluated by comparing the instant values of the Rayleigh number with the minimal values of the critical Rayleigh number [26,27]. The Rayleigh number was calculated according to the difference $\Delta T \equiv |T_R - T_C|$ between the droplet center and surface instant temperatures. Physical properties of liquid were chosen according to the instant average temperature of droplet mass. The results of water droplet state change, carried out in wide range of boundary conditions, show that the conditions, necessary for natural liquid circulation to start, are changing significantly during the droplet phase transformations (Figs. 1 and 2). The dynamics of the Rayleigh number variation is defined by extreme curves. In the case of conductive heating of the droplet, one extreme point is observed in the extreme curves; in the case of radiant-conductive heating—two extreme points. In both cases of the droplet heating the most favorable conditions of liquid circulation to arise are formed during the initial variation of the droplet state change, when the entire energy submitted to the droplet from outside is heating it. Surface layers of the droplet are heated most intensively; the difference between surface and center temperatures significantly grows, while the droplet diameter changes insignificantly (Fig. 3). In this extreme situation the Archimedean forces are caused by the difference between temperatures of droplet surface and central layers, as the condition $T_R > T_C$ is valid. In this case the droplet is heated by entire energy that is absorbed in it and the main part of external conductive heating energy, supplied to the droplet. After intensive growth during the initial period of the droplet state change, the interphase contact temperature gradually levels out, the rate of surface layers heating slows down, but the central layers of the droplet warm more intensively, therefore energetic level of the above-mentioned layers become equal; thermoconvective stability of the droplet increases. In the case of conductive heating the droplet enters into a new isothermal state, characteristic for equilibrium evaporation, and during which the Rayleigh number assumes zero value (Figs. 1 and 2a). In the case of combined heating the initial (highly unsteady) and the final (equilibrium) modes of droplet evaporation are connected by the transient period of the droplet phase change, during which the center layers of the droplet warm up to higher temperature if compared to the surface layers. Therefore the Rayleigh number, as it passes through its zero value, begins to increase again and the conditions for the beginning of natural liquid circulation become more and more favorable. The Archimedean forces are invoked by the difference between temperatures of droplet surface and central layers, as the condition $T_C > T_R$ is valid. In this case the droplet is heated only by part of radiant energy, absorbed in it, and it is very important, to what temperature the central layers of the droplet will warm, till its state ensures the conditions of equilibrium-evaporating mode. For that it is
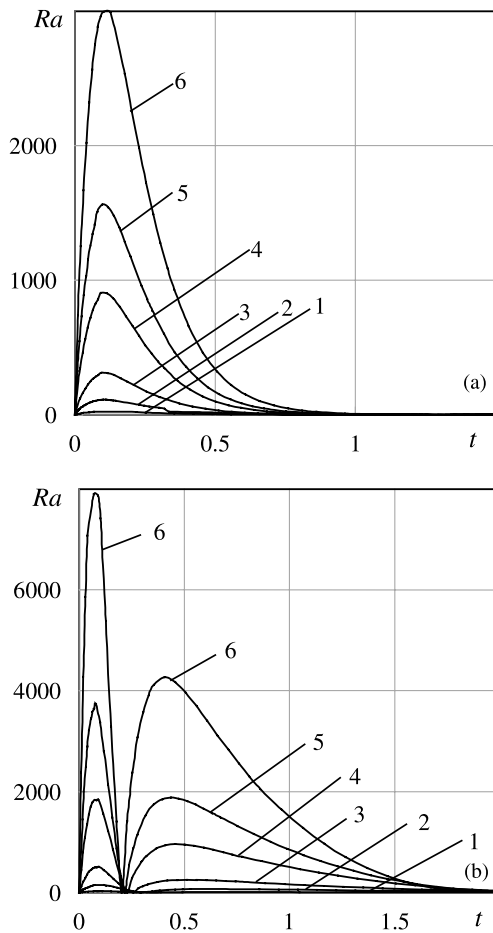
Fig. 1. Dependence of the Rayleigh number dynamics on the size of a water droplet in the case of convective (a) and radiant convective (b) droplet heating. $R_0$ (µm): (1) 150, (2) 250, (3) 350, (4) 500, (5) 600, (6) 750; $T_g = 1273$ K; $\Delta w = 0$.
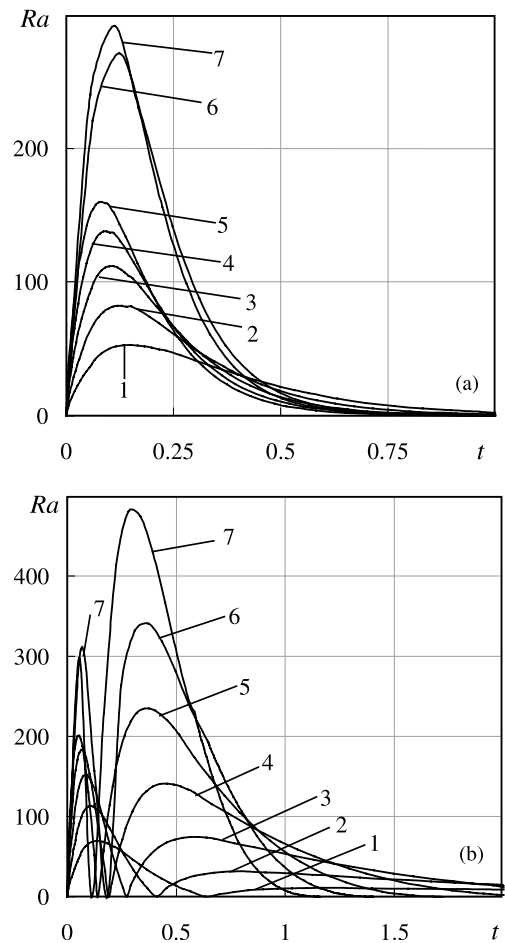
Fig. 2. Dependence of the Rayleigh number dynamics on gas temperature in the case of convective (a) and radiant convective (b) heating of a water droplet. $T_g$ (K): (1) 873, (2) 1073, (3) 1273, (4) 1473, (5) 1673, (6) 1873, (7) 2073; $R_0 = 250$ µm; $\Delta w = 0$.

necessary that the temperature field gradient inside the droplet would ensure the bringing out of all absorbed in the droplet radiant energy into its surface by conduction as well as by convection, if liquid circulation is present. Hence, the change of the droplet state during the transient period will be conditioned by the peculiarities of an external source and by nature of droplet liquid. The results of a numerical experiment prove that in the case of a conventional external radiation source, as hot gas environment, heated up to even 2000 K, radiates the droplets, the Archimedean forces are insufficient to cause liquid circulation in the droplets, the initial diameter of which is smaller than 1000 µm (Fig. 2b). In the final period of the droplet phase change an intensive equilibrium evaporation process occurs; the diameter of the droplet decreases rapidly and its thermoconvective resistance increases again.

In the case of combined heating of semitransparent liquid droplets, it is necessary to evaluate additionally the fact that if power of radiation source is high enough, the internal layers of the droplet can overheat to the temperature, higher than the saturated liquid temperature. In such case an explosive mode of droplet evaporation is possible. Experiments prove that the explosions of water droplets occur, when their mass mean temperature exceeds the temperature of liquid saturated state in 18 K [47]. In all modeled cases of power of a conventional external radiation source the above-mentioned conditions do not occur in evaporating water droplets (Fig. 4). Though one should remember that in the case of a powerful source of monochromatic radiation such mode of semitransparent liquid droplet evaporation is possible [48] and is well investigated theoretically and experimentally.
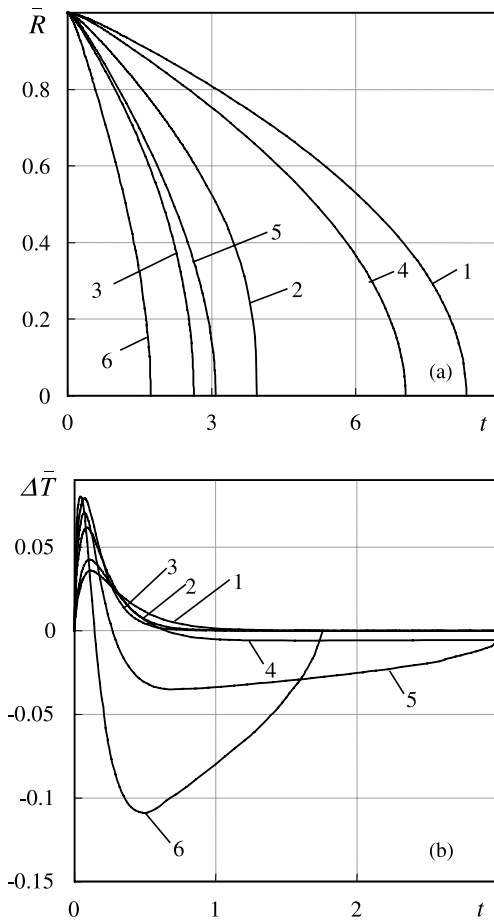
Fig. 3. Dynamics of evaporation (a) and non-isothermality (b) of a water droplet in the case of conductive (1–3) and radiant-conductive (4–6) droplet heating. $T_g$ (K): (1,4) 873, (2,5) 1273, (3,6) 1673, $R_0 = 250$ μm; $\overline{R} = R/R_0$; $\Delta \overline{T} = \overline{T}_R - \overline{T}_C$.
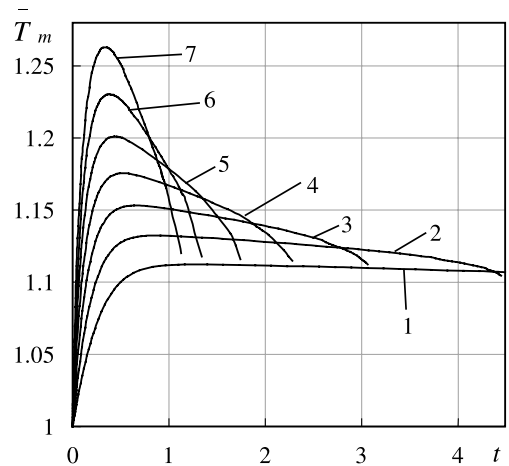


Fig. 4. Dynamics of mean mass of an evaporating water droplet in the case of radiant-conductive heating. $T_g$ (K): (1) 873, (2) 1073, (3) 1273, (4) 1473, (5) 1673, (6) 1873, (7) 2073; $R_0 = 250$ μm.

Performed thorough analysis of the state change of evaporating semitransparent liquid droplets in various boundary conditions allows to state that when the temperature of radiating gas flow $T_g < 2000$ K and the initial droplet diameter $2R_0 < 1000$ μm, then $f_{c,L}(\eta,t)|_{\eta=1} = 1$, when $\Delta w_L = 0$ and $f_{c,L}(\eta,t)|_{\eta=1} = k_c^-$, when $|\Delta w_L| > 0$.

### 4.2. The influence of droplet initial non-isothermality on its state change

When a droplet of non-isothermal initial state gets into a gas flow, we have: $\overline{T}_0(\eta,0) = \overline{T}_0(\eta)$ and $d\overline{T}_0(\eta)/d\eta \neq 0$. Though the last member of the expression (32) evaluates the intensity of energy (supplied to a droplet) consumption for the droplet phase transformations, i.e. allows calculating the instant temperatures in a droplet with changing diameter, but the expression

(13), which defines unsteady temperature field in a droplet of non-isothermal state, can be used only when droplet dimensions are constant. Therefore, the numerical schemes (34) and (37) are coordinated only in the case, when a surface mass source exists, which generates liquid proportionally to the evaporation rate. Obviously, a continuous scheme of the numeric investigation of combined transfer processes can be used for as long time period as you want only in a hypothetical droplet. Though the results of such investigations have not only cognitive value, but practical as well: they allow emphasizing the influence of the initial droplet non-isothermality on the course of droplet state change, as the influence of the droplet size variation is completely eliminated. Suppose that the initial temperature field in a spherically symmetrical droplet is defined by its coordinate function:

$$\overline{T}_{0,j} = \overline{T}_{0,J} + \Delta \overline{T}_0 (1 - \eta_j)^{n_1}, \tag{40}$$

here $\Delta \overline{T}_0 = \overline{T}_{0,C} - \overline{T}_{0,R}$. When $\Delta \overline{T}_0$ is positive, the droplet has been cooling prior to getting into air flow; the gradient of the initial temperature field of the droplet is negative. On the contrary, when $\Delta \overline{T}_0$ is negative, the droplet has been warming prior to getting into air flow; the gradient of the initial temperature field is positive. Semitransparent liquid droplets absorb radiant energy by their entire volume. Therefore, radiant energy, absorbed in the droplet, acts on its state as if the internal energy sources were existing in it. In order to purify the influence of the initial droplet non-isothermality on its heating dynamics and on new settled state of equilibrium evaporation of the constant size droplet, the effect of radiation is being cancelled ($q_{r,i,j} = 0$). The dependence of the transfer process parameters on the initial

state of evaporating semitransparent liquid was investigated by modeling the change of state of 200 µm diameter water droplet, heating and evaporating in air at 1273 K temperature. The initial temperature field of the droplet is defined by the expression (40), as $n_1 = 0.333$, $\Delta \overline{T}_0 = \pm 0.0666$. Boundary case of convective heating was modeled, as $\Delta w_0 = 0$, therefore $f_{c,L}(\eta, t)|_{\eta=1} = 1$.

The intensity of heating of a constant size droplet gradually decreases (Fig. 5a) and during equilibrium evaporation settles at the same value, without reference to the degree of the initial non-isothermality. It is conditioned by the peculiarities of the variation of the constant size droplet evaporation intensity (Fig. 5b). These peculiarities define the change of the function $f_i(B)$. This change is identical to dynamics of the ratio of the Nusselt numbers of evaporating and non-evaporating spheres: $f_i(B) = Nu_{L,i}^+/Nu_i$. Vapor flux on the droplet surface depends on liquid nature, droplet size, interphase contact surface temperature and gas flow para-
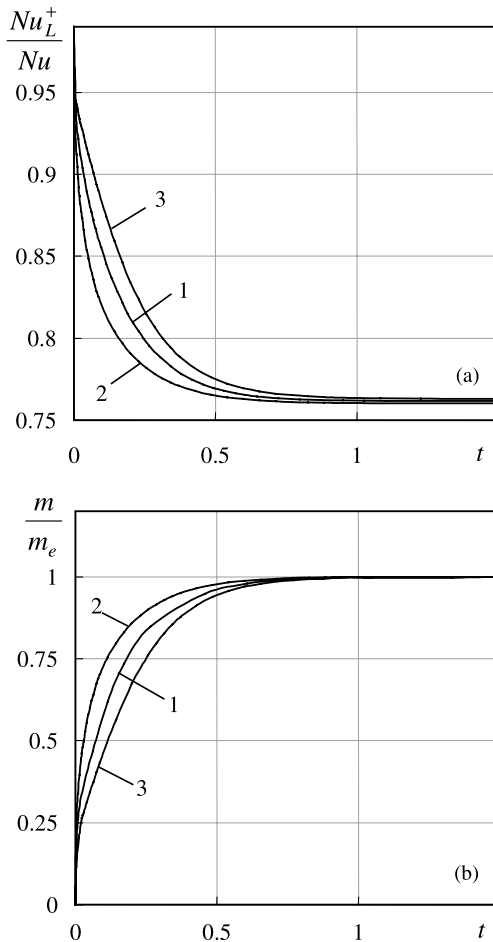


Fig. 5. Dependence of the intensities of a constant-size droplet conductive heating (a) and evaporation (b) on its initial state. $\Delta \overline{T}_0$: (1) 0, (2) 0.0666, (3) −0.0666.
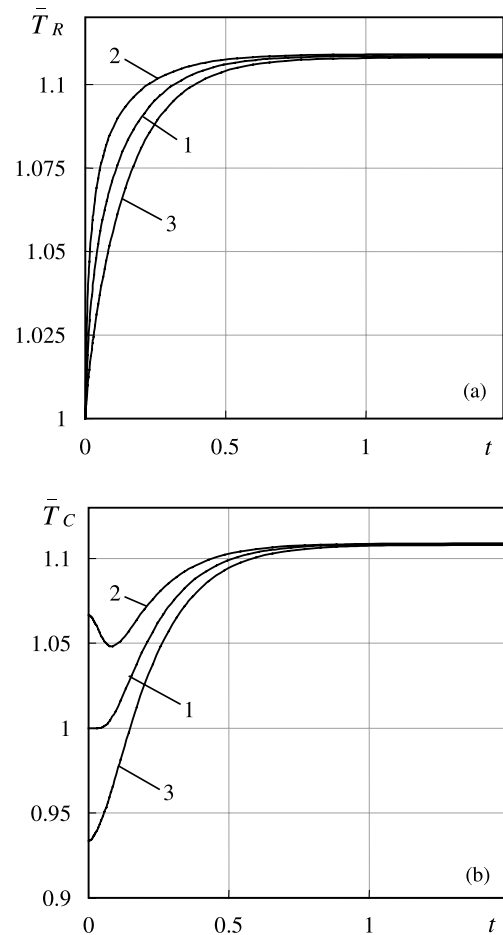


Fig. 6. Dependence of dynamics of an evaporating droplet surface (a) and center (b) temperatures on its initial state. $\Delta \overline{T}_0$: (1) 0, (2) 0.0666, (3) −0.0666.

meters. The latter do not change in the case of single droplet evaporation. Therefore the intensity of evaporation of constant size warming droplet is decided by dynamics of its surface temperature variation (Fig. 6). The temperature mode of an evaporating droplet depends on its initial state. In the case of positive $\Delta T_0$, initial enthalpy of the droplet is higher, if comparing it to the case of negative $\Delta T_0$. Therefore, as the initial conditions of external heating are similar, the instant temperature of the droplet layers is higher in the case of positive $\Delta T_0$ (Fig. 6). When the droplet surface temperature is higher, more intensive evaporation takes place (Fig. 5b, curve 2), therefore its influence on the heating intensity is more significant (Fig. 5a, curve 2).

The character of the rate of the droplet surface temperature variation confirms that the surface layers of the droplet warm in slowing rate in all cases of the initial non-isothermality (Fig. 7a). It is interesting, that the rate of heating of the droplet surface layers at the beginning
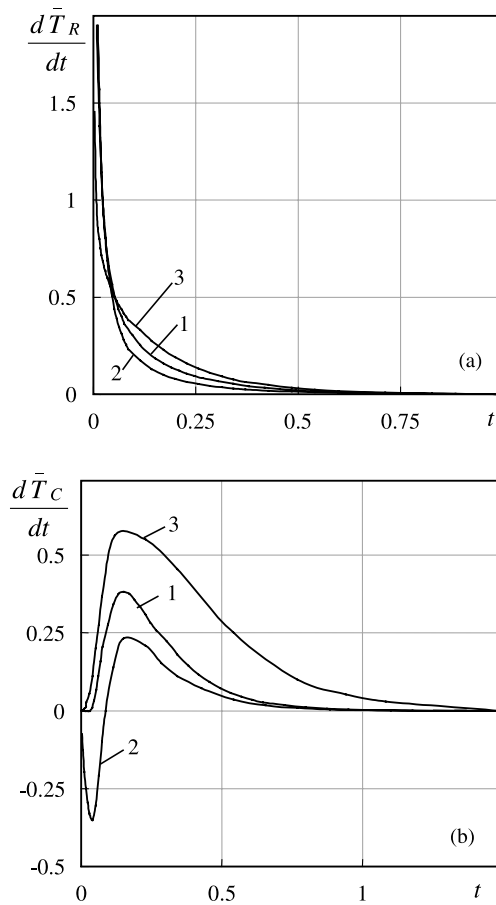
Fig. 7. The influence of a droplet initial state on the rate of heating of its surface (a) and central (b) layers. $\Delta \overline{T}_0$: (1) 0, (2) 0.0666, (3) −0.0666.
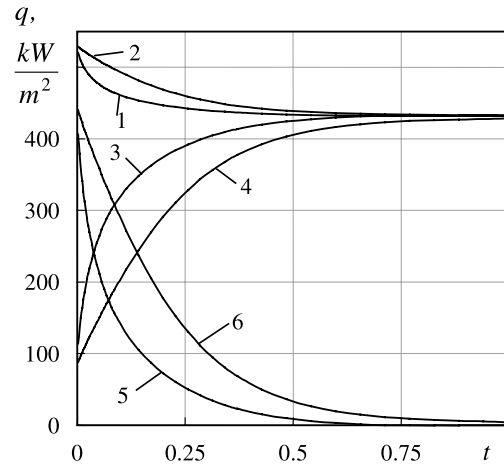


Fig. 8. The influence of a droplet initial state on the dynamics of heat fluxes on the droplet surface. $\Delta \overline{T}_0$: (1,3,5) 0.0666, (2,4,6) −0.0666; $q$: (1,2) $q_c^+$; (3,4) $q_f^+$; (5,6) $q_l^-$.

is higher in the case of positive $\Delta T_0$, later—in the case of negative $\Delta T_0$. This fact can be explained in the following way: when $\Delta T_0$ is positive, the droplet surface layers heat intensively and quicker reach the temperature (which does not depend on the degree of initial non-isothermality), characteristic to the isothermal state of equilibrium evaporation. The rate of surface layer heating, as it approaches equilibrium state, slows down more rapidly if compared to the case of negative $\Delta T_0$. Therefore, the curves of the droplet surface temperature variation cross with one another (Fig. 7a). Dynamics of the droplet center layer heating is more sensitive to the change of the initial droplet non-isothermality, if compared to the surface layers. As $\Delta T_0$ is changing, even qualitative changes of the rate of central layers are observed (Fig. 7b). The curves that define dynamics of the droplet center layer heating are extreme; they all have maximum point, and in the case of positive $\Delta T_0$ they have minimum point as well. When the initial temperature field gradient is negative, central layers of the

droplet are cooling for a while (Fig. 7b, curve 2), though its surface layers meanwhile are intensively warming.

Energy fluxes on the interphase contact surface are defined by the interaction of transfer processes in a droplet and its surroundings. The initial state of the droplet influences the dynamics of droplet heating and evaporation, therefore the distribution of energy supplied to the droplet between parts using for heating and evaporation, depends on the initial droplet state (Fig. 8). As the droplet heats, its temperature increases, the difference between temperatures $T_g$ and $T_R$ decreases, furthermore, the intensity of convective heating decreases, therefore convective heat flux decreases as well. In the case of positive $\Delta T_0$ the intensity of droplet evaporation is higher if compared to negative $\Delta T_0$, therefore when $\Delta T_0$ is positive, the decrease of convective heat flux is more rapid. At the beginning droplet heating is highly intensive, therefore the curves of conductive heat flux (5,6) are above the curves that show energy flux used for evaporation (3,4) (Fig. 8). Gradually bigger and bigger part of energy that externally heats the droplet is being involved into the evaporation process, therefore after some period of time the energy fluxes that heat and evaporate the droplet become equal. From this moment the curves of phase transformation flux are above the curves of conductive heat flux. After some time droplet surface temperatures settles down, and after that, as temperature of inner layers of the droplet settles down as well (Fig. 9), the equilibrium mode of the droplet evaporation begins. During this mode entire energy supplied to the droplets is involved into the evaporation process, therefore $q_l^- = 0$ and $q_f = q_c^+$. Hence, in the case of convective heating, without reference to the peculiarities of the initial droplet non-isothermality, which highly influence the transient temperature mode of the
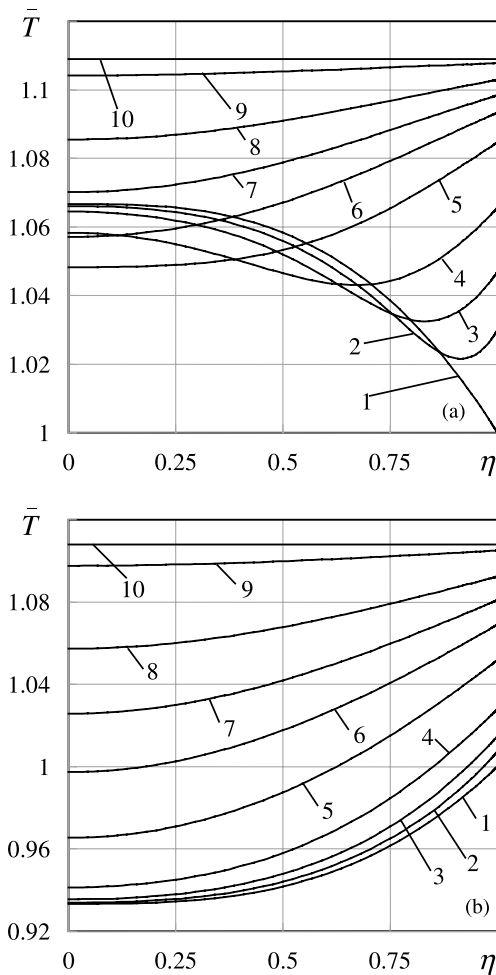
Fig. 9. Dependence of a droplet initial state on the droplet heating dynamics. $\Delta\overline{T}_0$: (a) 0.0666, (b) −0.0666; droplet heating time $t$: (1) 0, (2) 0.006, (3) 0.014, (4) 0.034, (5) 0.086, (6) 0.14, (7) 0.2, (8) 0.28, (9) 0.54, (10) 1.4.

droplet, it will reach the equilibrium evaporation mode being isothermal and at the same energetic level again. That corresponds to the physical interpretation of the droplet problem according to the given conditions and indirectly confirms reliability of the intended research method of unsteady transfer processes in liquid droplets under real conditions of combined heat and mass transfer.

### 4.3. Peculiarities of unsteady transfer processes in evaporating liquid droplets

Nature of condensed discrete and carrying media and the peculiarities of heat and mass transfer conditions are the essential factors that decide the regularities of unsteady transfer processes in the two-phase systems "droplets-gas". The peculiarities of heat and mass transfer conditions are in large measure conditioned by quantity of sprayed liquid, which is defined by the ratio of gas and liquid initial flow rates $\overline{G}_0 = G_{L,0}/G_{g,0}$, the spraying dispersity and the initial temperature of liquid. These peculiarities also depend on the parameters of carrying medium: gas temperature, velocity and volumetric part of liquid vapor in gas. $\overline{G}_0$ defines the degree of the influence of evaporated liquid on the change of gas flow state. When a single droplet evaporates in gas flow, it is stated that $\overline{G}_0 = 0$, and its heating and evaporation does not make any influence on the gas state. It is the simplest case of the two-phase system "droplets-gas", but its research allows emphasizing the peculiarities of the transfer processes in liquid droplets, fully eliminating the influence of gas flow state on the intensity of the transfer processes.

The investigations of droplet thermoconvective resistance, presented in Section 4.1 show that the peculiarities of radiation absorption in droplets make the essential influence on the intensity of the transfer process interaction in semitransparent droplets. Formally, taking into account the condition of energy flux balance on the droplet surface (32), radiation has no direct influence on the intensity of energy fluxes on the evaporating droplet surface, and consequently on the surface temperature as well. Clear defined and fully understandable influence will be made only by radiant energy, which corresponds to the intervals of radiation spectrum, in which the values of absorption coefficient are very large. The intensity of this radiation is expressed by the difference of radiant fluxes $q_r^+$ and $q_r^-$. Therefore, the expression (4) can be concretized in more general form:

$$q_c^+(\tau) + [q_r^+(\tau) - q_r^-(\tau)] - q_c^-(\tau) - q_f^+(\tau) = 0. \qquad (41)$$

The absorption coefficients of various semitransparent liquids have finite values in the entire radiation spectrum (Fig. 10), therefore the influence of the second member of the expression (41) is negligible, but indirect influence of radiation on the temperature of semitransparent liquid droplet surface is very big. This influence evidences as a result of the interaction of radiation and other ways of heat transfer in droplets. This interaction distorts the unsteady temperature field of the droplet, changes local temperature field gradient, hence conductive and convective heat fluxes are changed as well. The changes of temperature field in their turn evoke the change of liquid spectral optical characteristics; the conditions of light reflection on the internal and external sides of the droplet are changed (Fig. 11). The whole of these effects can condition up to 20% of the change of radiant flux in heating water droplets [14]. The intervals of the most intensive changes of spectral optical properties of semitransparent liquids mostly often coincide with the maximum values of the Planck function, therefore
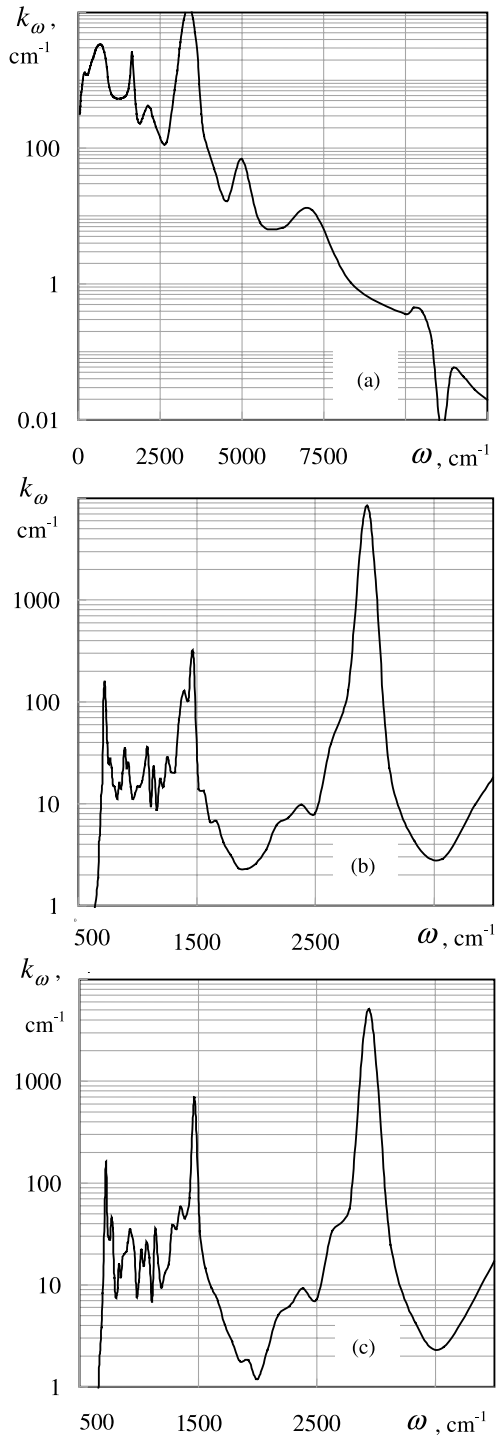
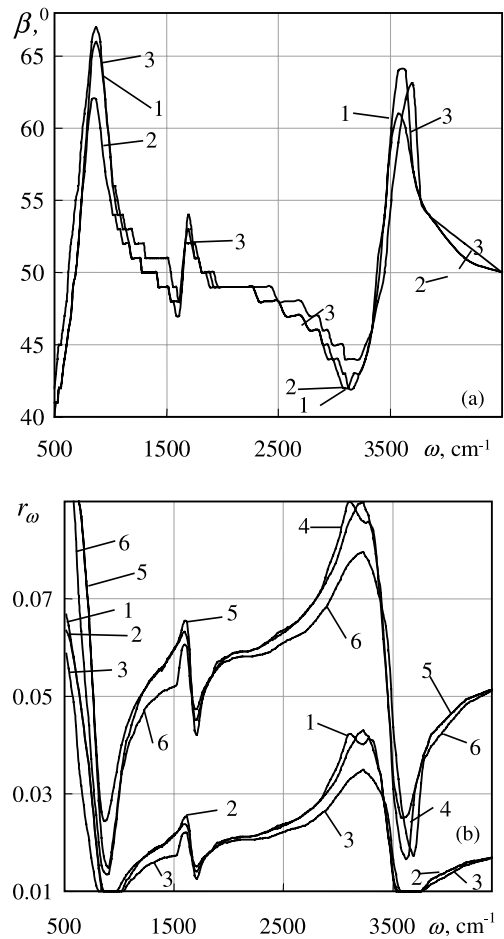Fig. 10. Spectral absorption coefficients of water (a), *n*-decane (b) and *n*-heptane (c).



Fig. 11. Dependence of the Brewsters angle (a) and light reflection index (b) on water temperature. $T_L$ (K): (1,4) 278, (2,5) 298, (3,6) 348; $\varphi$ (°): (1–3) 30, (4–6) 60.

significant qualitative changes of the radiation absorption in semitransparent liquids are being observed,

during the change of the temperature of radiant environment around the droplet. In such way, as the temperature of radiation source increases, water droplets become optically "more transparent". The intensity of radiation absorption becomes less dependent on the droplet size, therefore the family of curves, which describes the change of local radiant flux in the droplets, becomes significantly more concise, if compared to the case of the low temperature source (Fig. 12). The change of the spectral optical properties of *n*-decane and *n*-heptane is very much different from the change of water properties. The intensity of radiation absorption in the droplets of the above-mentioned liquids is less dependent on the changes of surrounding environment temperature (Fig. 13) if compared to water droplets, but it has its individual peculiarities.

The results of transfer process interaction in semi-transparent liquid droplets directly depend on the
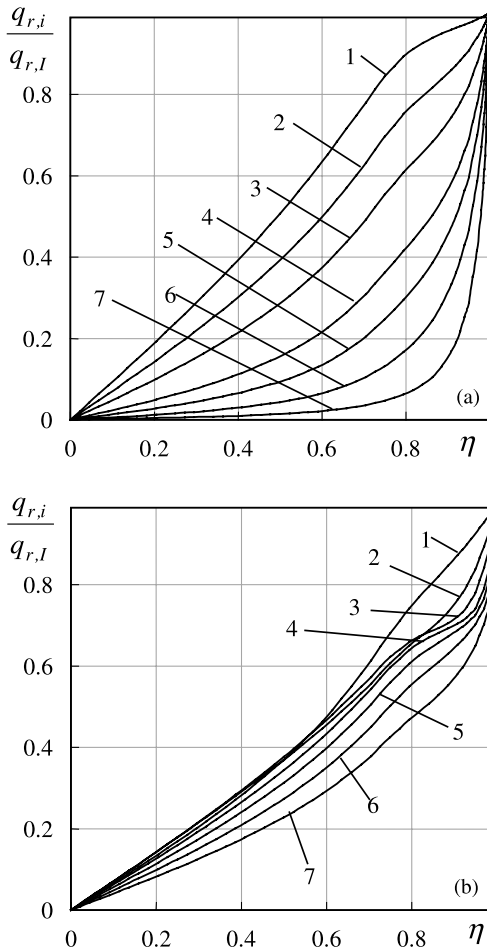
Fig. 12. The influence of radiant source temperature and droplet size on the intensity of radiant absorption. $T_g$ (K): (a) 673, (b) 1873; $R_0$ (μm): (1) 10, (2) 25, (3) 50, (4) 100, (5) 150, (6) 250, (7) 500; $T_L(\eta) = 298$ K.
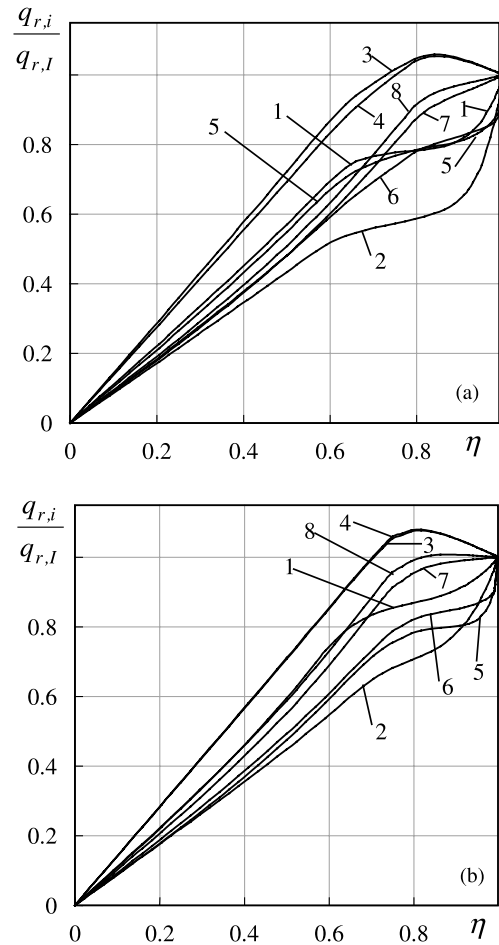
Fig. 13. Peculiarities of radiant absorption in n-decane (a) and n-heptane (b) droplets. $T_g$ (K): (1,5) 673, (2,6) 1073, (3,7) 1473, (4,8) 1873; $R_0$ (μm): (1–4) 25, (5–8) 500; $T_L(\eta) = 298$ K.

magnitude of radiation input in the droplet energy balance. Therefore the ratio of radiant and convective components of the total heat flux supplied to the droplet is very important parameter. This ratio significantly changes during the evaporation process (Fig. 14a). At the initial stage of intensive heating of the droplet the ratio increases as the convective heat flux decreases due to decrease of the temperature difference $T_g - T_R(t)$; and the radiant flux almost remains constant due to the insignificant change of the evaporating droplet diameter. The intensity of phase transformations increases when approaching to the equilibrium-evaporating mode (Fig. 15); therefore the droplet diameter changes significantly (Fig. 16). The change of the droplet diameter acts on the intensities of the total heat flux components in the different way: as the droplet becomes smaller, the flux of its convective heating increases; radiant flux—increases,

therefore the ratio $q_r^+(t)/q_c^+(t)$ very quickly decreases at the final stage of the droplet evaporation. Furthermore, the convective heat flux significantly depends on the intensity of the transfer process interaction in the droplet surroundings. The interaction of radiation and convective heat transfer is negligible, since the interaction of these two transfer processes in gas is much weaker than in semitransparent liquid. Therefore the influence of the Stefan flow is essential. Paradoxally, but as higher the intensity of convective heating is, the droplet will evaporate more intensively, and relative decrease of the convective heating intensity will be higher.

The change of the droplet surface temperature during its evaporation process significantly depends on the peculiarities of heat transfer between the droplet and its surrounding medium (Fig. 17). At the initial stage of unsteady evaporation droplet surface temperature
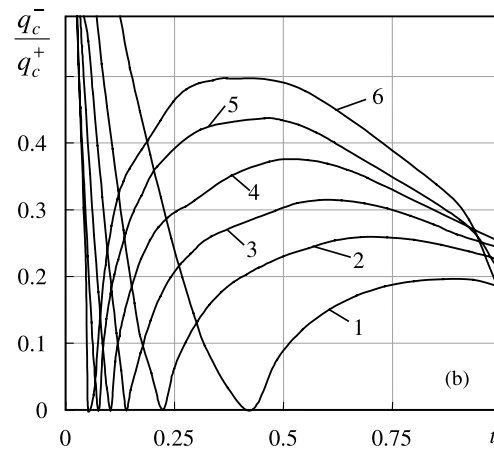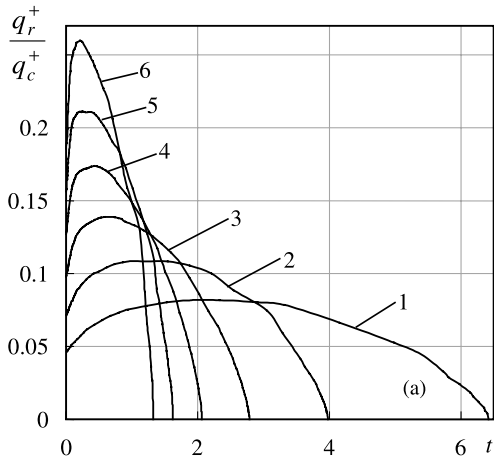
Fig. 14. Variation of heat flux ratios $q_r^+/q_c^+$ (a) and $q_c^-/q_c^+$ (b) during water droplet evaporation. $T_g$ (K): (1) 873, (2) 1073, (3) 1273, (4) 1473, (5) 1673, (6) 1873; $R_0 = 100$ μm.
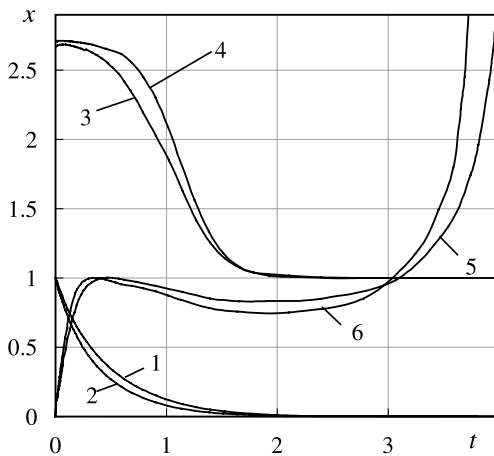


Fig. 15. The influence of velocity of water slip in gas on the intensity of liquid circulation and evaporation. Meaning of the parameter $x$: (1,2) $\Delta w/\Delta w_0$, (3,4) $k_c^-$, (5,6) $m/m_e$; $\Delta w_0$ (m/s): (1,3,5) 10, (2,4,6) 20; $R_0 = 100$ μm; $T_g = 1073$ K.
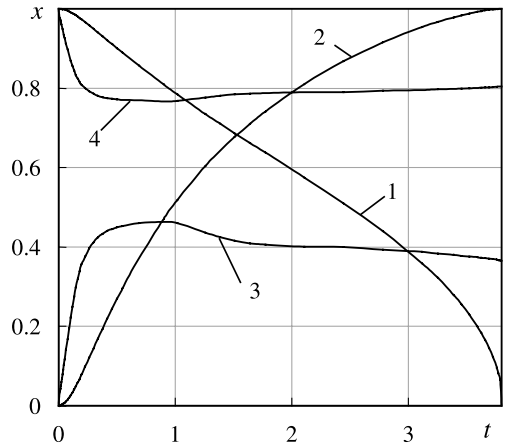


Fig. 16. Dynamics of an evaporating water droplet, evaporated liquid mass and the parameters that decide convective heating intensity. Meaning of the parameter $x$: (1) $R/R_0$, (2) $(1 - H/H_0)$; (3) $B$, (4) $f(B)$; $\Delta w_0 = 20$ m/s; $R_0 = 100$ μm; $T_g = 1073$ K.
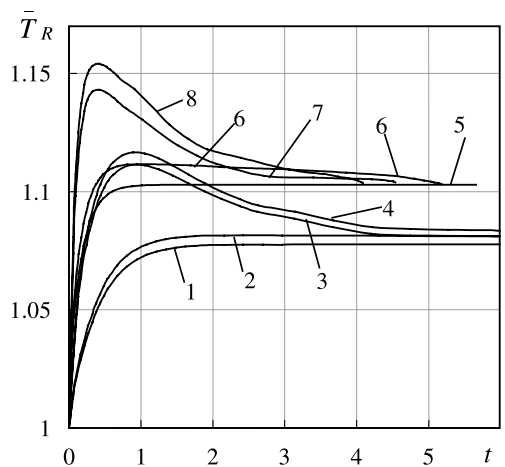


Fig. 17. Dependence of an evaporating droplet surface temperature variation on the way of droplet heating. The way of droplet heating: (1,5) conductive, (2,6) radiant-conductive, (3,7) convective, (4,8) radiant-convective; $T_g$ (K): (1–4) 673, (5–8) 1073; $\Delta w_0$ (m/s): (1,2,5,6) 0; (3,4,7,8) 10; $R_0 = 100$ μm.

increases. A single droplet reaches an equilibrium evaporation mode being already maximally warmed up. Though during equilibrium evaporation the droplet will be isothermal only in the case if energy is submitted to it by conduction. Then the droplet temperature will remain constant during the entire period from the very beginning of equilibrium evaporation till the moment of the droplet extinction: $T_{R,e,l} = $ const. This temperature is the function $T_{R,e,l} = f(T_g, p_{v,\infty})$ of the parameters of environment that surrounds the droplet. This function is independent of the initial state of the droplet and it corresponds to the lowest temperature of the droplet

surface, at which an equilibrium droplet evaporation can occur at given $T_g$ and $p_{v,\infty}$. The function $f(T_g, p_{v,\infty})$ will assume the lowest possible values in the case of droplet evaporation in dry air (Fig. 18a), when $p_{v,\infty} = 0$, therefore $T_{R,e,l} = f(T_g)$. Note that temperature $T_{R,e,l}$ is very sensitive to the ratio of pressures $p_{v,\infty}/p$ (Fig. 18b). As the latter increases, the evaporating droplet can warm up even more in low-temperature environment if compared to high-temperature environment, in which the ratio $p_{v,\infty}/p$ is smaller. In the cases of combined heating the droplet will reach an equilibrium evaporation mode having higher temperature than $T_{R,e,l}$. The increase of the droplet surface temperature, if compared to the case of conductive heating, will be conditioned by the ratios of the total heat flux: in the case of convective heating $\Delta T_{c,e} = T_{R,c,e} - T_{R,l,e} = f(q_{c,e}^+/q_{l,e}^+)$; in the case of radiant-

conductive heating $\Delta T_{r-l,e} = T_{R,r-l,e} - T_{R,l,e} = f(q_{r,e}^+/q_{l,e}^+)$; in the case of radiant-convective heating $\Delta T_{r-c,e} = T_{R,r-c,e} - T_{R,l,e} = f(q_{c,e}^+/q_{l,e}^+; q_{r,e}^+/q_{c,e}^+)$. The ratios of heat fluxes are changing during the equilibrium droplet evaporation as well; therefore the deviation of the droplet surface temperature from the temperature $T_{R,e,l}$ is not constant as well. $[\Delta T_{r-c,e}]_{R \to 0} \to 0$, because $[q_{c,e}^+/q_{l,e}^+]_{R \to 0} \to 1$ and $[q_{r,e}^+/q_{c,e}^+]_{R \to 0} \to 0$. Hence, with no reference to the peculiarities of heat transfer between a droplet and its surroundings, at the final evaporation stage the droplet temperature is close to the temperature $T_{R,e,l}$ (Fig. 19).

The ratio $q_c^-/q_c^+$ is important when evaluating the intensity of the interaction of convective heat transfer and mass transfer processes during the process of the droplet unsteady evaporation (Fig. 14b). The dynamics of the convective component $q_c^-$ of the total heat flux $q_\Sigma$ is very important for the dynamics of heat transfer parameter $B$, hence for the dynamics of the function $f(B)$ as well (Fig. 16). The physical meaning of $q_c^-$ changes during droplet heating and evaporation process. While the gradient of temperature field in the droplet is positive $(\partial T/\partial r|_{r=R-} > 0)$, $q_c^-$ corresponds to that part of the external convective heat flux $q_c^+$ that heats the droplet together with $q_r^-$. When the gradient of temperature field in the droplet becomes negative due to the interaction of transfer processes $(\partial T/\partial r|_{r=R-} < 0)$, $q_c^-$ corresponds to that part of the absorbed in the droplet radiant flux, which is taken into the droplet surface by convection and which evaporates the droplet together with $q_c^+$. Entire radiant energy, which has been absorbed in the droplet during the equilibrium evaporation mode, is
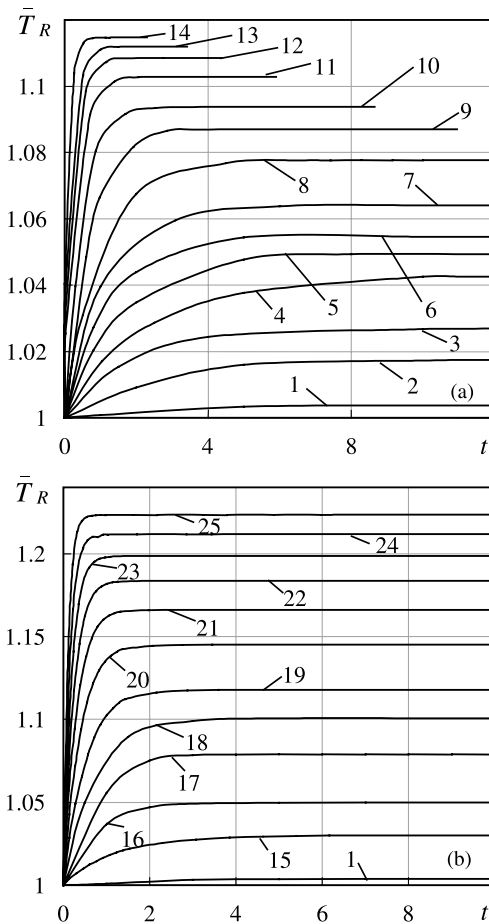


Fig. 18. Variation of an evaporating droplet surface temperature in the case of its conductive heating. $T_g$ (K): (1,15–25) 373, (2) 400, (3) 423, (4) 473, (5) 500, (6) 523, (7) 573, (8) 673, (9) 773, (10) 873, (11) 1073, (12) 1273, (13) 1473, (14) 1873; $p_{v,\infty}/p$: (1–14) 0, (15) 0.025, (16) 0.05, (17) 0.1, (18) 0.15, (19) 0.2, (20) 0.3, (21) 0.4, (22) 0.5, (23) 0.6, (24) 0.7, (25) 0.8, (26) 0.9; $R_0 = 50$ μm.
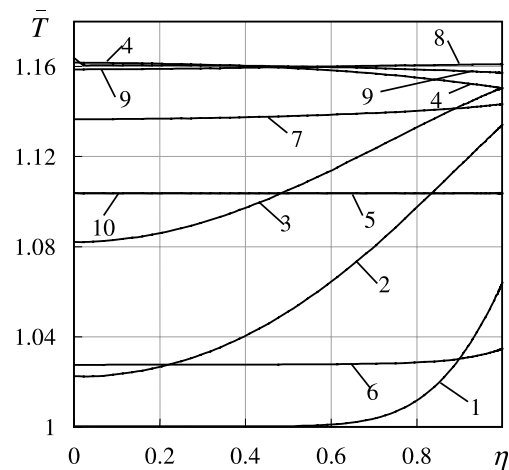


Fig. 19. The influence of liquid circulation on heating of an evaporating droplet. The way of calculation: (1–5) according to the model of radiant-conductive heat transfer, (6–10) according to the model of radiant-convective heat transfer; $t$: (1,6) 0.019, (2,7) 0.118, (3,8) 0.195, (4,9) 0.59, (5,10) 3.8; $R_0 = 100$ μm; $T_g = 1073$ K; $\Delta w_0 = 20$ m/s.

taken into the droplet surface by convection, therefore $q_c^- \equiv q_r^-$. Total heat flux, which defines the intensity of the droplet heating, equals zero ($q_\Sigma^- = 0$), therefore the ratios $q_c^-/q_c^+$ and $q_r^-/q_c^+$ are identical during the process of the equilibrium droplet evaporation (Fig. 14).

The change of velocity of droplet slip in gas has a major influence on heat convection in an evaporating droplet (Fig. 15). The intensity of convection is defined by a parameter $k_c^-$. Higher is the initial velocity of the droplet slip in gas $\Delta w_0$, more intensive is its relative decrease (Fig. 15, curves 1,2), but the influence of liquid circulation on heat transfer in the droplet is bigger (Fig. 15, curves 3,4). The droplet velocity gradually approaches to the velocity of gas due to drag, therefore the ratio $[\Delta w/\Delta w_0] \to 0$; the parameter $k_c^- \to 1$, i.e. radiant-conductive heat transfer prevails in the droplet for about a half of evaporation time, despite of the fact that $\Delta w_0 > 0$. Hence, when $\Delta w_0 > 0$, liquid circulation plays a critical role on the droplet state during the entire unsteady evaporation mode and at the initial stage of equilibrium droplet evaporation (Fig. 15). Droplet evaporation intensity, defined by the ratio of densities of vapor flow on the droplet surface $m/m_e$, constantly grows during the unsteady transfer mode (Fig. 15, curves 5,6), as the droplet surface temperature increases. At the initial stage of the unsteady transfer mode the ratio $m/m_e$ decreases for some time as the droplet surface cools (due to reduction of the ratio $q_{c,e}^+/q_{l,e}^+$); later on, at the final evaporation stage ($R \to 0$), it begins to increase very quickly, as the intensity of conductive heating $q_{l,e}^+$ increases. The duration of unsteady evaporation significantly depends on the initial droplet state. The influence of the unsteady evaporation mode depends on the difference between the initial droplet temperature $T_{R,0}$ and the temperature $T_{R,e}$, which is characteristic to the moment of unsteady evaporation beginning, i.e. how far the ratio of these temperatures $T_{R,e}/T_{R,0}$ is from 1. Note that despite of the fact that the equilibrium evaporation mode is much longer than the unsteady one, the influence of the latter is quire significant, because during the unsteady evaporation mode the droplets loose up to 20% of liquid mass (Fig. 16).

The experimental results of temperatures and phase transformation rates of evaporating droplets show that evaporation dynamics and the temperature mode of an evaporating droplet significantly depend on liquid nature and the boundary conditions of heat and mass transfer [2,5,9,41,45–50]. Evaporation of water droplets is investigated thoroughly. Please note that the results of the experimental measurements of the temperatures of equilibrium evaporation mode are scattered in quite wide range (Fig. 20, points). Though these results in wide range of gas temperature variation can be quite well defined theoretically according to the model of transfer process in droplets, presented in this study; by changing pressure ratio $p_{v,\infty}/p$ (Fig. 20, lines). The ex-
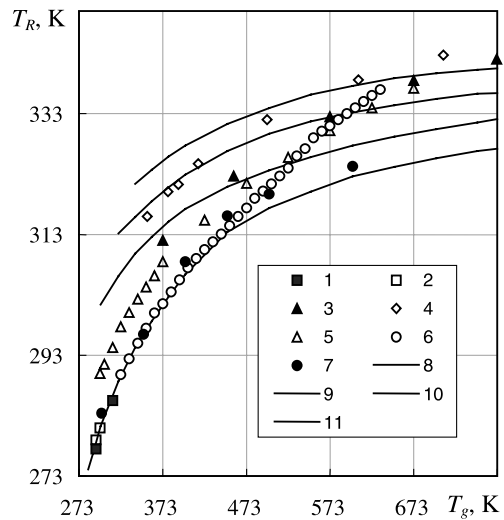


Fig. 20. Temperature of an evaporating water droplet. Points—the experiment [9,49]: (1) G. Langstrofh, (2) W. Ranz, W. Marschall, (3) A. Kobaijisi, (4) N. Nishiaki, (5) V. Fedoseev, D. Poliscuk, (6) M. Apasev, R. Malov, (7) C. Downing; lines—numerical investigation, when $\Delta w = 0$, $p_{v,\infty}/p$: (8) 0, (9) 0.05, (10) 0.1, (11) 0.1.

perimental results of large water droplet evaporation under the influence of radiation from hot surfaces and high temperature dry air, presented in the study [50], prove a significant impact of radiation on the interaction of transfer processes in evaporating water droplets, as well as on the rate of their phase transformations. The experiments were performed with 900–2900 μm size water droplets, evaporating in equilibrium conditions in dry air at temperature 673–1073 K. This was achieved by a continuous pumping out of water vapor in velocity, not higher than 0.01 m/s. The constancy of droplet size was ensured by a special capillary system. The authors stated that maximal error of droplet evaporation rate did not exceed 5%. The results of these experiments are quite well described by the model of radiant-conductive droplet heating, presented in this study, stating that the source of radiation is absolutely black body, the temperature of which equals to that of air, as $p_{v,\infty}/p = 0$ (Fig. 21, lines). By the way, this premise explains theoretically obtained evaporation rate, which is a little bit higher, because in the case of more rigorous investigations the emissivity of grey surfaces should be evaluated, which has not been done in the study [50].

## 5. Conclusions

Combined transfer processes in droplets and in their surroundings are strictly related. Their regularities can be investigated only by performing thorough integrated
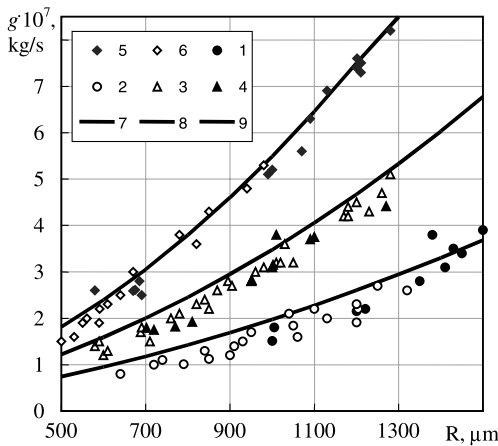
Fig. 21. The rate of water droplet evaporation. Points—the experiment (V. Ivanov, E. Smirnova) [50]; lines—numerical investigation; $T_g$ (K): (1,2,7) 818, (3,4,8) 981, (5,6,9) 1133.

researches of the transfer process interaction. The peculiarities of radiant absorption in semitransparent liquids have a decisive influence on the interaction of unsteady transfer processes in evaporating droplets; the Stefan flow has a significant influence on the transfer processes in droplet surroundings. The intensity of convective heat transfer depends on the velocity of droplet slip in gas, which evokes a forced circulation inside the droplets. The conditions of natural heat convection to begin are changing during the droplet evaporation, but they are insufficient to start liquid circulation because of the Archimedean forces, as the droplets evaporate in the boundary conditions of heat and mass transfer, which are characteristic to usual thermal technologies. The initial state of droplets also influences their heating and evaporation. The conditions of heat transfer during the droplet heating and evaporation are always changing, their peculiarities are important for the dynamics of the droplet state. The ratio $q_c^-/q_c^+$ is important when evaluating the intensity of interaction of convective heat transfer and mass transfer processes during the process of droplet unsteady evaporation. The dynamics of the convective component $q_c^-$ of the total heat flux $q_\Sigma^-$ is very important for the interaction of convective heat and mass transfer in the droplet surroundings. As $\partial T/\partial r|_{r=R^-} > 0$, $q_c^-$ corresponds to that part of the external convective heat flux $q_c^+$ that heats the droplet together with $q_r^-$. As $\partial T/\partial r|_{r=R^-} < 0$, $q_c^-$ corresponds to that part of $q_r^-$, which evaporates the droplet together with $q_c^+$. During equilibrium evaporation $q_c^- \equiv q_r^-$. In the case of conductive heat transfer to the droplet, the droplet will reach the equilibrium evaporation mode being least heated. This temperature does not depend on the droplet initial state and is the function of the surrounding medium para-

meters $T_g$ and $p_{v,\infty}$. In the case of combined heating of the droplet the ratios of heat fluxes $q_{c,e}^+/q_{l,e}^+$ and $q_{r,e}^+/q_{c,e}^+$ have an additional influence on the degree of the droplet heating.

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